

UNPUBLISHED PRELIMINARY DATA

Space Technology Project No. 4

THE SCATTERING OF PULSED LASER LIGHT

FROM PLASMAS

13P.
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Introduction

In principle, light scattering from a plasma can provide valuable diagnostic information on parameters which are usually only measured approximately through the use of probes, whose disturbance of the plasma must be taken into account. Hot plasmas are luminescent sources and are also relatively poor scatterers of light; the order of magnitude of the basic cross section is provided by the effective area of an electron, whose "radius" is of the order of 10^{-13} cm., leading to a cross section of the order of 10^{-26} cm.² per electron. Nevertheless, the availability of intense monochromatic sources provided by pulsed lasers makes it feasible to attempt to observe both the angular distribution and spectral distribution of laser light scattered from plasmas of relatively modest densities and temperatures; plasmas which can be produced with relative ease. For these plasmas the Debye length $\lambda_D = \sqrt{kT/4\pi n_0 e^2}$ is considerably longer than the 6943 angstrom wavelength of the ruby laser radiation and the principal effect will be scattering from the independent particle motion rather than the collective

modes and fluctuations of the plasma, however, collective effects can still be measured by observing the small angle scattering.

Scattering Theory

The scattering of light from a free electron is governed by the Thompson scattering cross section. In dealing with a plasma the coulomb interaction between the particles makes them less independent and tends to reduce their scattering ability. The correlations of positions of electrons and ions determine the general nature of the scattering and these correlations in turn describe density fluctuations in the plasma.

Consider a single scatterer located at r_j scattering an incident wave with propagation vector K_o . The asymptotic form of the scattered wave is

$$\phi(r) = e^{iK_o \cdot r} - \frac{a_j e^{iK \cdot r}}{r} e^{i(K_o - K) \cdot r_j}$$

The effective scattering amplitude is $a_j e^{iK \cdot r_j}$ where $K = K_o - K$. If there are many particles, the amplitude is the sum $A = \sum_{j=1}^N a_j e^{iK \cdot r_j}$, and the cross section is $|A|^2 = \sum_{ij} a_i a_j e^{iK \cdot r_{ij}} = |a|^2 \sum_{ij} e^{iK \cdot r_{ij}}$ if $a_i = a_j = a$. If the particles are free to move, their position vectors must be regarded as dynamical variables and the amplitude now is the matrix element between an initial state $|i\rangle$ and a final state $|f\rangle$ of the scattering medium, i.e. $\langle f | |a|^2 \sum_{ij} e^{iK \cdot r_{ij}} | i \rangle$

Neither the initial state of the system nor the final state are observed. Instead we average over a suitable chosen ensemble of initial states and sum over all final states compatible with the conservation of energy. The energy transferred to the scattering system is $E = K\Delta\omega = h(\omega_0 - \omega^1)$, where ω^1 and ω_0 are respectively the frequencies of the scattered light and incident light. We require $E_i - E_f = E$. The cross section contribution becomes:

$$\int_{h\omega_0}^{\infty} dE \sum_f |\langle f | A | i \rangle|^2 \delta(E - E_i + E_f)$$

After substituting an integral representation of the delta function and using the Heisenberg representation for the dynamical variables, we obtain

$$\int e^{i\Delta\omega t} \langle i | A(0) A^\dagger(t) | i \rangle dt$$

which must be averaged over the ensemble of initial states; the ensembles being defined by the density matrix ρ , we have $\bar{A} = \text{tr } \rho A$; and

$$|a|^2 \int e^{i\Delta\omega t} \sum_{ij} \overline{e^{iK \cdot [r_i(0) - r_j(t)]}} dt$$

$$\text{We now define the density operator } n(r, t) = \sum_i \overline{\delta[r - r_i(t)]}$$

$$\text{and the two point density operator } n(r, t; r', t') = \sum_{ij} \overline{\delta[r - r_i(t)] \delta[r' - r_j(t')]}.$$

It follows that

$$\begin{aligned} \int n(r, t, r + \Delta r, t') dr &= \int n(r) n(00 | \Delta r, t - t') dr \\ &= \int \sum_{ij} \overline{\delta[r - r_i(t)] \delta[r + \Delta r - r_j(t')]} dr \end{aligned}$$

where $n(0|r)$ is conditional density function. Therefore we have

$$n(0|\Delta r, t-t') = \frac{1}{N} \int \sum_{ij} \delta[r - r_i(t)] \delta[r + \Delta r - r_j(t')] dr$$

The terms with $i = j$ represent the single particle contributions, while the terms with $i \neq j$ are the two particle contribution.

Using these functions we can write

$$\sum_{ij} e^{iK \cdot [r_i(0) - r_j(t)]} = N \int e^{-iK \cdot r} n(0|rt) dr$$

and the scattering cross section per unit solid angle per unit energy transfer becomes

$$\frac{d^2 \sigma}{d\Omega d\epsilon} = |a|^2 \int e^{i(\Delta\omega t - K \cdot r)} n(0|rt) dr dt$$

Finally introducing the specific form for a^2 for unpolarized waves, and defining the dynamic form factor $S(K, \Delta\omega)$ by the integral, we have

$$\frac{d^2 \sigma}{d\Omega d\epsilon} = \left(\frac{e^2}{mc^2} \right)^2 (1 - 1/2 \sin^2 \theta) S(K, \omega)$$

A central problem in plasma theory is the calculation of the dynamic form factor. We discuss here briefly some simple results.

A. Electron Gas

A classical electron gas in thermal equilibrium with a smeared out positive charge background has a dynamics form factor.

$$S(K, \omega) = \frac{N}{K} \frac{f(\omega/k)}{1 + 4\pi\alpha(K, \omega)}$$

where $f(u) = \left(\frac{m}{2\pi KT}\right)^{1/2} e^{-\frac{mu^2}{2KT}}$ and α is the polarizability. If the Debye wavelength is much larger than the light wavelength, the polarizability is negligible and the form factor becomes

$$S(K, \omega) = \frac{N}{K} f\left(\frac{\omega}{K}\right)$$

which is the individual particle spectrum. If the Debye wavelength is less than the light wavelength the collective modes contribute to the scattering through plasma waves and

$$S(K, \omega) = \frac{N}{K} \frac{f(\omega/K)}{1 - (\omega/\omega_p)^2 + i\pi \omega/K \lambda^2/\lambda_D^2} f(\omega/K)$$

leading to maxima in the scattering when

$$\omega = \pm \omega_p = \pm \sqrt{\frac{4\pi Ne^2}{m}}$$

B. Electron and Ion Plasma with Equal Temperature

The form factor has two terms

$$S(K, \omega) = \frac{N}{K} \left[f_{el}\left(\frac{\omega}{K}\right) \left| \frac{1}{1 + 4\pi\alpha_{el}} \right|^2 + f_{ion}\left(\frac{\omega}{K}\right) \left| \frac{4\pi\alpha_{el}}{1 + 4\pi\alpha_{ion} + 4\pi\alpha_{el}} \right|^2 \right]$$

The spectrum of scattered radiation consists of the doppler broadened principal line characteristic of the single particle motions and sharp maxima at

$$+ \Delta\omega = \pm \omega_p = \sqrt{\frac{4\pi N e^2}{m}}$$

provided that $K\lambda_D \ll 1$. Since $K = \frac{2\omega_0}{c} \sin \frac{\theta}{2}$, small angle scattering would reveal this structure, arising from collective motions.

Finally to be noted is the occurrence of maxima in the form factor associated with the onset of instability. This phenomenon is similar to critical opalescence. Experiments which are able to detect the angular and frequency distributions of the scattered light would allow us to confirm experimentally much of the above predictions and provide valuable diagnostic information on plasma parameters.

Design of Experiment to Study Free Particle Scattering Modes.

When the source wavelength is much smaller than λ_D and the scattering is observed at large angles, the principal contributions to the dynamic form factor arises from the terms with $i = j$ in $n(00|rt)$. For this case we have

$$n(00|rt) = \frac{1}{(2\pi)^3 N} \int e^{iK \cdot r} \overline{\sum_j e^{iK \cdot [r_j(0) - r_j(t)]}}_{dk}.$$

If the particles are free, their equation of motion is

$$\dot{P}(t) = 0 \quad \dot{r}(t) = \frac{P(0)}{M} \quad r(t) = r(0) + t/M P(0)$$

Recalling the commutation relation $[r_i(0), P(0)] = i\hbar\delta_{ij}$ we obtain

$$\begin{aligned} n(00|rt) &= \left(\frac{1}{2\pi}\right)^3 \int e^{iK \cdot (r - \frac{pt}{M})} e^{i\hbar K^2 t / 2M} dk \\ &= \left(\frac{iM}{2\pi\hbar t}\right)^{3/2} e^{i\left[-\frac{iM}{2\pi\hbar t} \left(r - \frac{pt}{M}\right)^2\right]}. \end{aligned}$$

Averaging over a Maxwellian distribution and passing to the classical limit $\hbar \rightarrow 0$, we obtain

$$n(00|rt) = \left(\frac{M}{2\pi t^2 KT} \right)^{3/2} e^{-\frac{Mr^2}{2Kt^2}}$$

Taking the Fourier transform, we obtain the dynamic form factor

$$S(K, \Delta\omega) = \left(\frac{2\pi M}{KTK} \right)^{1/2} e^{-\left(\frac{M\Delta\omega}{2KTK^2} \right)^2}.$$

Recalling that $\Delta\omega$ is the frequency shift and K is the change in momentum of the scattered radiation the exponential can be expressed as $\exp - \left(\frac{\Delta\lambda}{\lambda} \right)^2 \frac{Mc^2}{4KT}$, showing that the Maxwellian distribution of the electrons is reproduced in the intensity of the scattered light. Plotting the logarithm of the signal amplitude against $(\Delta\lambda)^2$ will give the electron temperature.

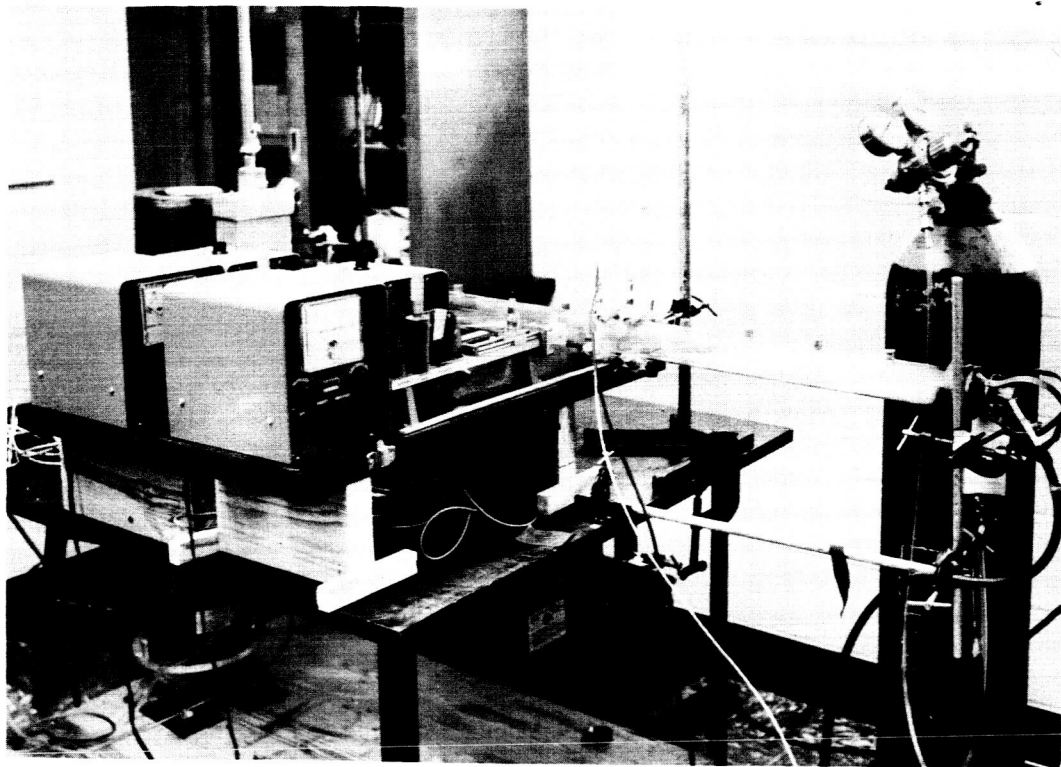
We next estimate the intensity of the scattered radiation for the experiment which is being set up. The plasma parameters are assumed to be

$$N_e = 10^{12} \text{ electrons/cm}^3$$

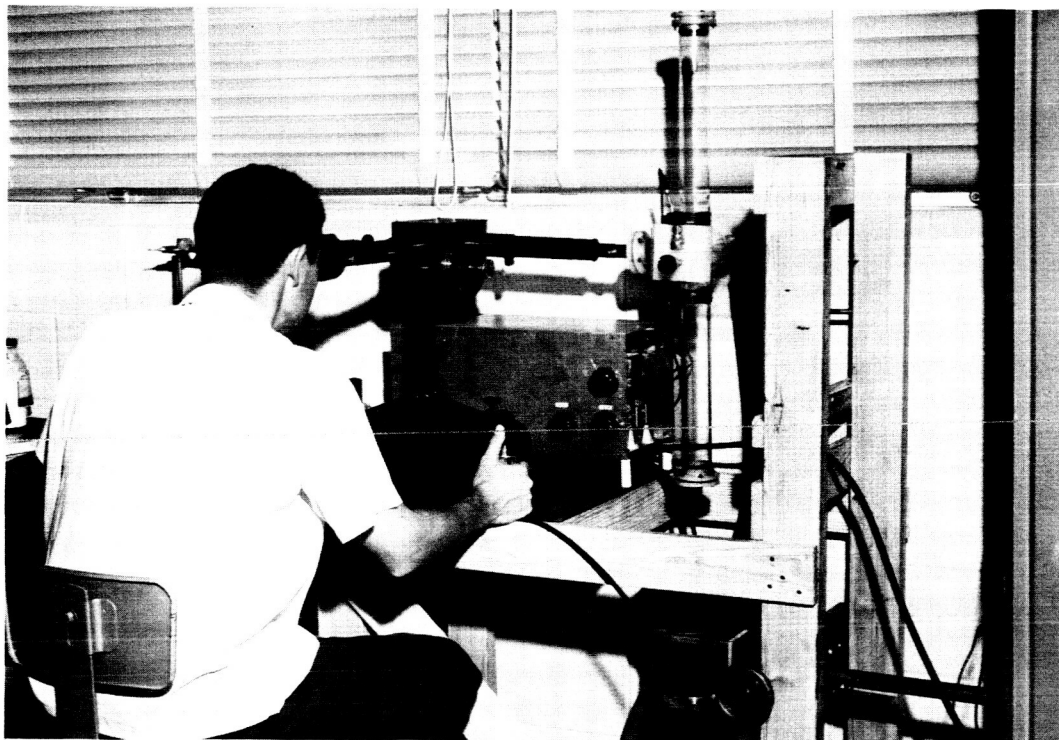
$$T_e = 10^2 \text{ e.v.},$$

while the laser beam is assumed to contain 0.1 joule lasting for 1 millisecond. The laser is focused to a 0.1 cm^2 cross section and scattered radiation is collected from 2 cm. of path length.

The total scattered power at 90° is 1.6×10^{-11} watts/steradian, which leads to 5.6×10^4 photons scattered into a steradian. Assuming about $1/4$ of a steradian is intercepted by the viewing lens, we have 1.4×10^4 photons



Microwave diagnostics of plasma produced in electrically driven shock tube.



Spectroscopic observation of laser scattering from R. F. plasma.

per laser flash entering the spectrometer. The spectrometer is blazed for 7000°A, so that approximately 10^4 photons can be expected to be found on the spectrometer exit slit, contained within a band of about 120 angstroms, centered at 6943 angstroms. There will be about 50 photons per angstrom per flash. The spectrometer has a dispersion of 4 angstroms per mm. in the 2nd order, so that a 1 mm. slit should pass 200 photons per pulse onto the photo-multiplier. The power received by the photomultiplier

$$P = (200) (2.83 \times 10^{-19}) / 10^{-3} \sim 6 \times 10^{-14} \text{ watts.}$$

The photomultiplier currently being used is the 7102, which has an equivalent noise level at - 60° C of 5×10^{-14} watts. The signal to noise can be improved by sacrificing some resolution, however, a 9558 A photomultiplier can be used to improve the signal to noise by a factor of about 3. Further, taking into account the spiked nature of the laser pulse, the instantaneous scattering intensity should be considerably above the average level, improving the signal to noise. The experiment appears to be feasible.

Steady State Experiment

One experiment which is being conducted involves an attempt to observe the electron velocity distribution in a plasma produced by an R. F. discharge. A rough plan of the experiment is shown in figure 1. While the photograph shows the equipment in its actual form, the principal task remaining is the construction of adequate viewing dumps, required to

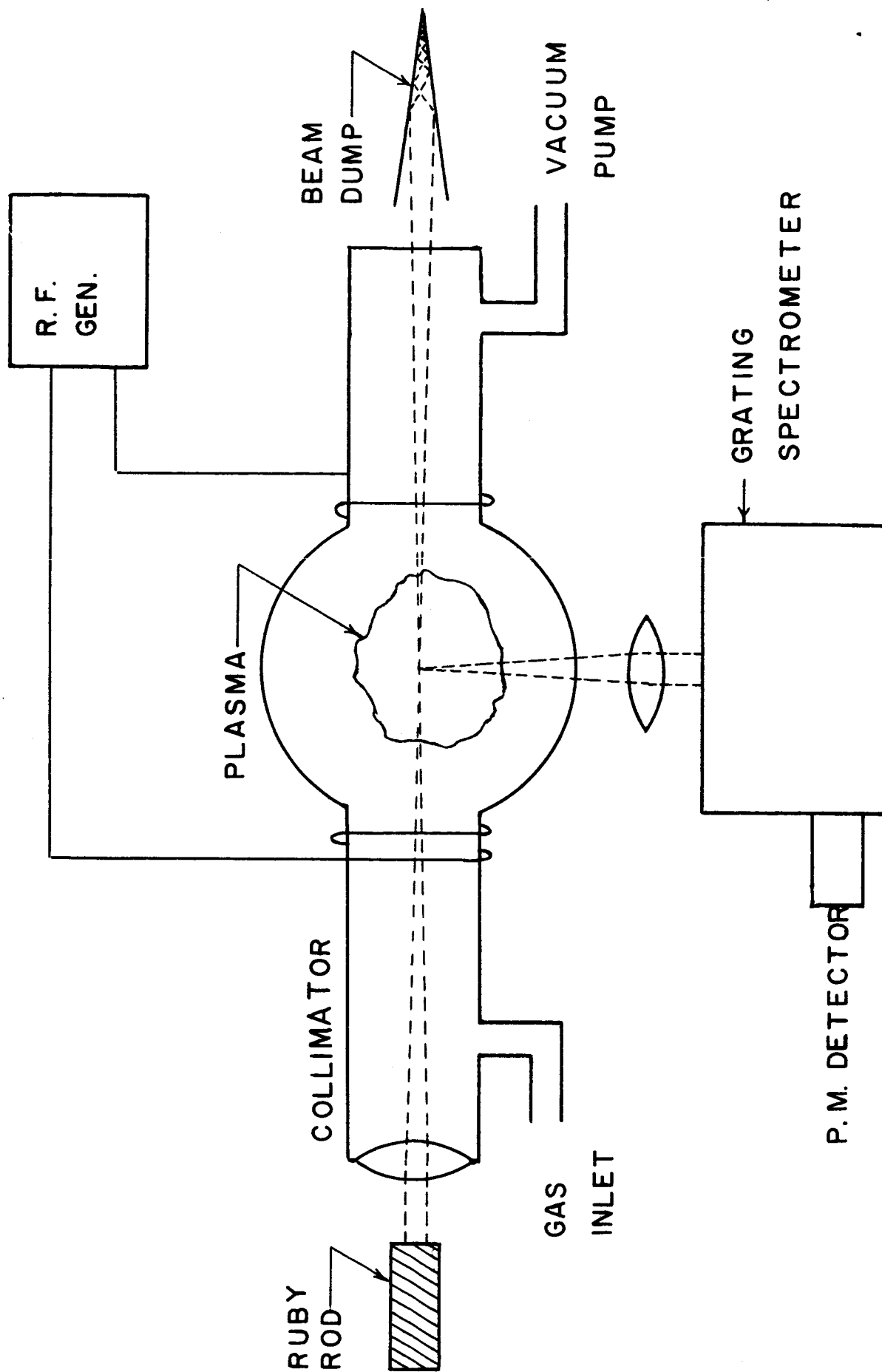


Figure 1.

attenuate the laser beam after scattering. Considering the tremendous difference between the incident power and the scattered power, it can readily be appreciated that primary light scattered from surface irregularities, which subsequently enters the detector, would mask the scattered light by a substantial factor. Although the spectral analysis affords some relief from the effects of non-plasma scattering, the intensity close to the central line is much more intense, making it very desirable to work as close to the central line as possible. Dumps employing highly polished carrera glass are now being constructed and it is hoped they will provide sufficient attenuation.

The principal progress has been made in completing the various oscillators, power supplies, photomultiplier detectors and other electronic systems. The scattering cell, vacuum system and laser have been successfully built and tested. It is hoped that a scattering experiment can be performed in the near future.

Transient Plasma Experiment

The purpose of this experiment is to study the scattering in transient plasmas where electron densities of the order of $10^{16}/\text{cm}^3$ are achieved. A plasma gun ionizes a puff of gas introduced through a fast acting gas valve and propels it with speeds of cm. per microseconds. The plasma is directed into a strong magnetic field produced by discharging a capacitor through the field coil. It is hoped that trapping for the order of several

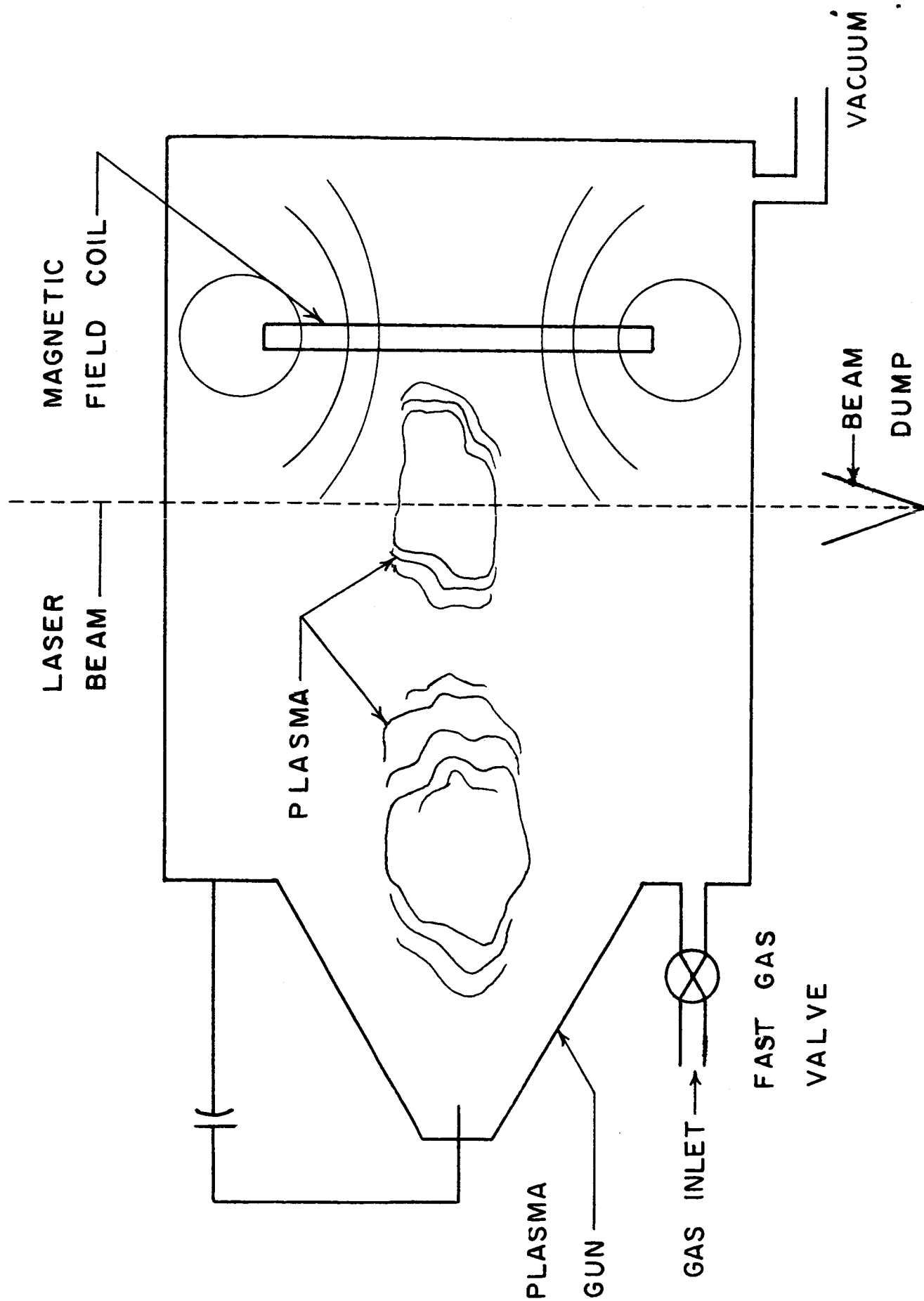


Figure 2.

microseconds can be achieved for a dense plasma. Figure 2 shows a rough schematic of the apparatus and the photograph shows the set up of a current experiment, where microwaves are being used to measure the velocity of the plasma puff, and estimating its density. The vacuum system, the plasma gun, the fast gas valve and their associated electronic circuits have been built and tested. It is hoped that a giant laser pulse can be obtained using a Kerr cell Q spoiler to obtain laser power whose duration is a better match to the plasma lifetime.

Summary

The program carried out during the past year has taken up to the point where most of the necessary equipment has been built and tested and a theoretical study of many of the pertinent areas has been carried out. It is expected that the experiments planned can be carried out in the near future.